5 SEM TDC MTH M 1

2021

(March)

MATHEMATICS

(Major)

Course : 501

(Logic and Combinatorics, and Analysis—III)

Full Marks : 80

Pass Marks : 32/24

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

(Logic and Combinatorics)

1. *(a)* State whether the given statement is *True* or *False* : 1

'If *P*, then *Q*' is also known as '*Q*, provided that *P*'.

(b) If the statements P, Q, R and S are assigned the truth values T, F, F and T respectively, then find the truth value of the following statement : 2

$$R \wedge S \rightarrow (P \rightarrow \sim Q \vee S)$$

(c) If $P \leftrightarrow Q$ is T, what can be said about the truth values of $P \leftrightarrow \sim Q$ and $\sim P \leftrightarrow Q$? 1+1=2

(d) What is a tautology? 1

(e) Construct the truth table of the following statement : 4

$$P \rightarrow (Q \rightarrow R)$$

Or

Prove that—

if $\vDash A$ and $\vDash A \rightarrow B$, then $\vDash B$

2. (a) Define a term. 1

(b) Translate the following in symbols : 1×2=2

(i) All freshmen are intelligent.

(ii) Some rationals are not reals.

(c) Find a formal derivation of

$A \to (B \to C),\ \sim D \vee A,\ B \vDash D \to C$ 3

(d) Derive mathematically any *one* of the following : 4

(i) All animals are mortal.
All human beings are animals.
Therefore, all human beings are mortal.

(ii) No human beings are quadrupeds.
All women are human beings.
Therefore, no women are quadrupeds.

3. (a) What is the value of

$$\sum_{r=0}^{n} C(n, r)\ ?$$ 1

(b) Find the coefficient of $x_1^3 x_2 x_3^2$ in the expansion of $(2x_1 - 3x_2 + 5x_3)^6$. 2

(c) Show that if m and n are integers greater than 1, then $R(m, n) \le C(m+n-2,\ m-1)$. 4

Or

For all integers n and r with $1 \leq r \leq n-1$, show that

$$C(n, r) = C(n-1, r) + C(n-1, r-1)$$

4. Answer any *two* of the following : 4×2=8

(a) Find the number of integers between 1 and 1000, inclusive, that are not divisible by 5, 6 and 8.

(b) Determine the generating function for the sequence of squares

$$0, 1, 4, \cdots, n^2, \cdots$$

(c) Solve :

$a_n = a_{n-2} + 4n$ with $a_0 = 3$, $a_1 = 2$ through a generating function.

GROUP—B

[Analysis—III (Complex Analysis)]

5. (a) What do you mean by singular point of a function? 1

(b) Write Cauchy-Riemann equations. 1

(c) If $f(z) = u + iv$ is an analytic function in domain D, prove that the curves $u = \text{constant}$, $v = \text{constant}$, form two orthogonal families. 3

(d) Prove that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, \; z \neq 0, \; f(0) = 0$$

is continuous and Cauchy-Riemann equations are satisfied at the origin yet $f'(z)$ does not exist at $z = 0$. 5

Or

If $u = x^3 - 3xy^2$, show that there exists a function $v(x, y)$ such that $w = u + iv$ is analytic in a finite region.

6. *(a)* Define Jordan arc. 1

(b) Verify Cauchy's theorem by integrating e^{iz} along the boundary of the triangle with vertices at the points $1 + i$, $-1 + i$ and $-1 - i$. 4

(c) If a function $f(z)$ is analytic within and on a closed contour C and a is any point lying in it, then prove that

$$f'(a) = \frac{1}{2\pi i} \int_C \frac{f(z)\,dz}{(z-a)^2}$$

5

(d) Evaluate

$$\int_C \frac{e^{2z}}{(z+1)^4}\,dz$$

where C is $|z|=3$. 4

Or

If $f(z)$ is a continuous function in a domain D and if for every closed contour C in the domain D

$$\int_C f(z)\,dz = 0$$

then prove that $f(z)$ is analytic within D.

7. (a) State and prove Taylor's series. 1+5=6

Or

Find Laurent's series about the indicated singularity of the function

$$f(z) = \frac{e^{2z}}{(z-1)^3},\ z=1$$

Name the type of singularity and give the region of convergence. 3+2+1=6

(b) Expand

$$\log\left(\frac{1+z}{1-z}\right)$$

in a Taylor's series about $z=0$. 2

8. *(a)* Define essential singularity of an analytic function $f(z)$. 1

(b) Find the singularities of

$$\frac{\cot(\pi z)}{(z-a)^2}$$

at $z=a$ and $z=\infty$. 2

(c) Evaluate any *two* of the following : 5×2=10

(i) $\int_0^{2\pi} \frac{\cos 2\theta}{5+4\cos\theta} d\theta$

(ii) $\int_0^{\infty} \frac{dx}{1+x^2}$

(iii) $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)^2}$

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