Total No. of Printed Pages—11

3 SEM TDC GEMT (CBCS) GE 3 (A/B/C)

2022

(Nov/Dec)

MATHEMATICS

(Generic Elective)

Paper : GE–3

Full Marks : 80

Pass Marks : 32

Time : 3 hours

The figures in the margin indicate full marks for the questions

Paper : GE–3A

(Real Analysis)

1. *(a)* Define countable set. 1

(b) Show that the set $\mathbb{Z}$ of all integers is denumerable. 3

(c) Show that if $ab > 0$, then *either (i)* $a > 0$ and $b > 0$ *or (ii)* $a < 0$ and $b < 0$. 2

(d) If $a \in \mathbb{R}$ is such that $0 \le a \le \varepsilon$ for every $\varepsilon > 0$, then show that $a = 0$. 2

(e) Prove that if $x \in \mathbb{R}$, then there exists a positive integer n such that $x \le n$. 4

Or

Prove that if x and y are real numbers with $x < y$, then there exists a rational number $r \in \mathbb{Q}$ such that $x < r < y$.

2. *(a)* Define an open interval. 1

(b) Show that if $y > 0$, then there exists $n_y \in \mathbb{N}$ such that $n_y - 1 \leq y \leq n_y$. 3

(c) Show that if $I_n = [a_n, b_n]$, $n \in \mathbb{N}$ is a nested sequence of closed, bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $\inf\{b_n - a_n : n \in \mathbb{N}\} = 0$, then the number ξ contained in I_n for all $n \in \mathbb{N}$ is unique. 4

Or

Prove that the set $\mathbb{R}$ of real numbers is not countable.

3. *(a)* Define limit of a sequence. 1

(b) Define bounded sequence. 1

(c) Prove that the sequence (n) is divergent. 2

(d) Prove any *one* of the following : 3

(i) $\lim\left(\dfrac{1}{n^2+1}\right) = 0$

(ii) $\lim\left(\dfrac{3n+2}{n+1}\right) = 3$

(e) Show that every convergent sequence of real numbers has a unique limit. 4

Or

Prove that a convergent sequence of real numbers is bounded.

4. (a) Define Cauchy sequence. 1

(b) Prove that every convergent sequence is a Cauchy sequence. 4

(c) Prove that every sequence of real numbers is convergent if and only if it is a Cauchy sequence. 4

Or

Prove that if (x_n) and (y_n) are convergent sequences of real numbers and if $x_n \leq y_n$ for all $n \in \mathbb{N}$, then $\lim(x_n) \leq \lim(y_n)$.

5. (a) Define alternating series. 1

(b) Prove that if the series Σx_n converges, then $\lim(x_n) = 0$. 2

(c) Prove that the series

$$\sum \frac{\sin nx}{n^2}$$

is absolutely convergent. 3

(d) Show that the series Σx_n converges if and only if for every $\varepsilon > 0$, there exists $M(\varepsilon) \in \mathbb{N}$ such that if $m > n \geq M(\varepsilon)$, then

$$|S_m - S_n| = |x_{n+1} + x_{n+2} + \ldots + x_m| < \varepsilon$$ 4

Or

Prove that the alternating series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$$

is convergent.

6. (a) Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}$$

is convergent. 5

Or

Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

is divergent.

(b) Test for convergence (any *one*) : 5

(i) $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots$ to ∞

(ii) $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \cdots$ to ∞

7. *(a)* Define limit of a sequence of functions. 1

(b) Write the statement of Weierstrass M-test. 2

(c) Prove that the sequence (f_n), where

$$f_n(x) = \frac{x}{n}, \; x \in \mathbb{R}$$

is pointwise convergent on $\mathbb{R}$. 3

(d) Prove that the sequence (f_n), where $f_n(x) = \frac{1}{x+n}$ is uniformly convergent on any interval $[0, b]$, $b > 0$. 4

8. *(a)* Define radius of convergence of a power series. 1

(b) If the radius of convergence of a power series is zero, then the series

(i) converges everywhere;

(ii) converges nowhere.

Write the correct answer. 1

(c) Prove that if R is the radius of convergence of $\Sigma a_n x^n$ and K be a closed and bounded interval contained in the interval of convergence $(-R, R)$, then the power series converges uniformly on K. 4

Or

Prove that a power series can be integrated term-by-term over any closed and bounded interval.

(d) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$, where (any *one*)

(i) $a_n = \dfrac{n^n}{n!}$

(ii) $a_n = \dfrac{(n!)^2}{(2n)!}$ 4

Paper : GE–3B

(Cryptography and Network Security)

1. *(a)* Distinguish between conventional and public-key cryptosystems. What are the basic requirements of a public-key cryptosystem? 3+3=6

(b) Explain active attack and passive attack with real-life examples. 3+3=6

(c) What is message authentication? Define the classes of message authentication function. What are the requirements for message authentication? 2+3+4=9

(d) Differentiate between MAC and Hash function. 6

2. Explain the Secure Hash Algorithm (SHA) with neat diagram. 10

Or

Illustrate MD5 algorithm in detail.

3. Write a note on any *one* of the following : 5

(a) DSS

(b) TCP session hijacking

(c) Teardrop attack

(d) SSL

4. Explain the architecture of IP security in detail. 8

Or

What are transport and tunnel modes in IPsec? Describe how ESP is applied to both these modes.

5. *(a)* Explain SNMP architecture in detail. 6

(b) What is firewall? Describe how firewall can be used to protect the network. 8

Or

Describe the working of Secure Electronic Transaction (SET) with neat diagram.

6. Write short notes on any *two* of the following : 8×2=16

(a) VPN

(b) Smurf attack

(c) Intrusion Detection System (IDS)

(d) Encapsulating Security Payload (ESP)

Paper : GE–3C

(Information Security)

1. Answer any *five* of the following questions : 2×5=10

(a) What is user authentication in information security?

(b) What is cryptography?

(c) Define virus.

(d) What are worms in terms of information security?

(e) What is cipher?

(f) How does a plain text differ from cipher text?

(g) What is a hash function?

2. *(a)* Compare and contrast protection and security. 3

(b) Briefly explain any *three* aspects of security from the following : 4×3=12

(i) Data availability

(ii) Privacy

(iii) Data integrity

(iv) Authentication

3. Briefly explain any *three* of the following : 5×3=15

(a) Trojan horse

(b) Trap door

(c) Stack

(d) Buffer flow

4. How do system threats differ from communication threats? Explain with examples. 4+6=10

5. (a) How does substitution cipher differ from transposition cipher? 5

(b) How does public-key cryptography differ from private-key cryptography? 5

Or

Briefly explain the functionalities of Data Encryption Standard (DES).

6. Briefly explain the functionalities of digital signatures. What is MAC? 8+2=10

7. Explain any *two* of the following : 5×2=10

(a) Intrusion detection

(b) Tripwire

(c) RSA algorithm

(d) Diffie-Hellman key exchange

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