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3 SEM TDC MTMH (CBCS) C 7

2022

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C–7

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. *(a)* Find the degree of the equation

$$x\frac{\partial^2 z}{\partial x^2}+y\left(\frac{\partial z}{\partial y}\right)^{1/3}+Kz=0$$ 1

(b) Define linear partial differential equation. 1

(c) Write the general form of Lagrange's equation. 1

(d) Form the PDE by eliminating the arbitrary functions f and ϕ from 5

$$z=yf(x)+x\phi(y)$$

Or

Solve :

$$(x^2-yz)p+(y^2-zx)q=z^2-xy$$

(e) Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ which passes through the curve $xz=a^3$, $y=0$. 5

Or

Solve :

$$\sqrt{p}+\sqrt{q}=1$$

2. (a) Write the Jacobi's subsidiary equations. 2

(b) Find the complete integral of any *one* of the following : 4

(i) $(p^2+q^2)y=qz$

(ii) $pxy+pq+qy=yz$

(iii) $p=(z+qy)^2$

(c) Find the complete integral of

$$p_3x_3(p_1+p_2)+x_1+x_2=0$$ 6

Or

Solve the boundary value problem $\frac{\partial u}{\partial x}-2\frac{\partial u}{\partial y}=u$ with $u(x,0)=6e^{-3x}$ by the method of separation of variables.

3. (a) Write the Laplace equation. 1

(b) Classify the following equations :

(i) $$(1-x^2)\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+(1-y^2)\frac{\partial^2 z}{\partial y^2}+2x\frac{\partial z}{\partial x}+6x^2y\frac{\partial z}{\partial y}-6z=0$$ 2

(ii) $u_{xx}+u_{yy}+u_{zz}+u_{yz}+u_{zy}=0$ 2

(c) Reduce the equation

$$y(x+y)(r-s)-xp-yq-z=0$$

to canonical form. 7

Or

Derive the one-dimensional wave equation.

4. (a) Fill in the blank :

The PDE in case of vibrating string problem is formulated from the law of _____. 1

(b) Write one-dimensional heat equation. 1

(c) Solve

$$\frac{\partial^2 u}{\partial x^2}-2\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}=0$$

using the method of separation of variables. 6

Or

Find the solution of $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}$ such that $y=p_0\cos pt$ where p_0 is constant when $x=l$ and $y=0$ when $x=0$.

5. (a) Give an example of a linear system of ordinary differential equation with variable coefficient. 1

(b) Transform the linear differential equation $\frac{d^3x}{dt^3}+2\frac{d^2x}{dt^2}-\frac{dx}{dt}-2x=e^{3t}$ into system of first order differential equation. 2

(c) Prove that $x=2e^t, y=-3e^{2t}$ is the solution of $\frac{dx}{dt}=5x+2y$, $\frac{dy}{dt}=3x+4y$. 2

(d) Describe the method of successive approximation. 4

Or

Find first two approximations of the function that approximate the exact solution of the equation $\frac{dy}{dx}=x+y$, $y(0)=1$.

(e) Find the general solution of the system :

$$\frac{dx}{dt}=x+2y,\ \frac{dy}{dt}=3x+2y$$ 6

Or

Using operator method, find the general solution of

$$\frac{dx}{dt}+\frac{dy}{dt}-2x-4y=e^t,\ \frac{dx}{dt}+\frac{dy}{dt}-y=e^{4t}$$

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