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**5 SEM TDC DSE MTH (CBCS)**  
**1.1/1.2/1.3 (H)**

**2 0 2 2**

( Nov/Dec )

**MATHEMATICS**

( Discipline Specific Elective )

( For Honours )

Paper : DSE-1

Full Marks : 80

Pass Marks : 32

Time : 3 hours

*The figures in the margin indicate full marks  
for the questions*

Paper : DSE-1.1

**( Analytical Geometry )**

1. Answer the following questions :

(a) Write the vertex of the conic

$$(x - 1)^2 = 2(y + 2) \quad 1$$

(b) Find the equation of the ellipse whose ends of major axis  $(0, \pm 6)$ , and passes through the point  $(-3, 2)$ . 4

( 2 )

- (c) Write the processes to sketch the ellipse. 4

- (d) Identify and sketch the curve

$$y^2 - 8x - 6y - 23 = 0$$

and also label the focus, vertex and directrix. 6

Or

Describe the graph of the hyperbola

$$16x^2 - y^2 - 32x - 6y - 57 = 0$$

and sketch its graph.

2. Answer the following questions :

- (a) Write the condition of tangency of the line  $y = mx + c$  to the parabola  $y^2 = 4ax$ . 1

- (b) Write the reflection property of ellipse. 1

- (c) Write the equation of the asymptotes of the hyperbola  $\frac{x^2}{4} - \frac{y^2}{9} = 1$ . 1

- (d) Derive the equation of tangent to the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at the point  $(x_1, y_1)$ . 6

- (e) Find the equation of the hyperbola whose length of transverse axis 7 units and foci  $(\pm 5, 0)$  and also sketch it. 6

Or

Find and sketch the curve of the ellipse whose foci  $(1, 2)$  and  $(-1, -2)$  and the sum of the distances from each point  $P(x, y)$  on the ellipse is 6 units.

3. Answer the following questions :

- (a) Write the condition that the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

represent a pair of straight lines. 1

- (b) Write the condition that the quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

represents an ellipse. 1

- (c) Determine a rotation angle  $\theta$  that will eliminate the  $xy$ -term of the conic

$$x^2 - 4xy + 4y^2 - 5 = 0$$

2

- (d) Show that the graph of the given equation

$$x^2 - 10\sqrt{3}xy + 11y^2 + 64 = 0$$

is a hyperbola. Find its foci, vertices and asymptotes.

5

- (e) Let an  $x'y'$ -coordinate system be obtained by rotating an  $xy$ -coordinate system through an angle  $\theta = 60^\circ$ .

(i) Find the  $x'y'$ -coordinate of the point whose  $xy$ -coordinate is  $(-2, 6)$ .

(ii) Find an equation of the curve  $\sqrt{3}xy + y^2 = 6$  in  $x'y'$ -coordinate.

6

Or

Identify and sketch the curve

$$9x^2 - 24xy + 16y^2 - 80x - 60y + 100 = 0$$

4. Answer the following questions :

(a) Write the equation of a sphere whose centre is at the origin and radius is  $r$ .

1

(b) Write True or False :

1

Curve of intersection of two spheres is a sphere.

(c) Write the standard equation of hyperbola of one sheet. 1

(d) Write the equation of the tangent plane to the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

at  $P(x_1, y_1, z_1)$ . 2

(e) Find the equation of the sphere passes through the points  $(0, 0, 0)$ ,  $(0, 1, -1)$ ,  $(-1, 2, 0)$ ,  $(1, 2, 3)$ . 5

(f) A sphere of constant radius  $k$  passes through the origin and meets axes in  $A$ ,  $B$  and  $C$ . Prove that the centroid of the triangle  $ABC$  lies on the sphere

$$9(x^2 + y^2 + z^2) = 4k^2 \quad 5$$

Or

Find the equation of the sphere whose centre at  $(1, 2, 3)$  and touching a plane at  $(2, 1, 3)$ .

5. Answer the following questions :

(a) Find the radius and centre of the circle

$$x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0, \quad x - 2y + 2z = 3$$

5

- (b) Find the equation of the sphere whose great circle is

$$x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, \quad x + y + z = 3 \quad 5$$

Or

Prove that the two spheres

$$x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$$

and  $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$

touch each other.

6. Answer the following questions :

- (a) Find the equation of the two tangent planes to the sphere

$$x^2 + y^2 + z^2 - 2y - 6z + 6 = 0$$

which are parallel to the plane

$$2x + 2y - z = 0 \quad 5$$

- (b) Classify and sketch the quadric surface (any one) : 5

(i)  $36x^2 + 9y^2 + 16z^2 = 144$

(ii)  $4x^2 - 3y^2 + 12z^2 + 12 = 0$

Paper : DSE-1.2

( Portfolio Optimization )

1. Answer any *five* of the following questions :

1×5=5

- (a) Why do individuals invest?
- (b) Write the formula for holding period return (HPR).
- (c) What is business risk?
- (d) What is security market line (SML)?
- (e) What is mutual fund?
- (f) Define diversification.

2. (a) If a person invests ₹ 200 at the beginning of the year and get back ₹ 220 at the end of the year, find the holding period return (HPR) and holding period yield (HPY) of the investment. 2+2=4

(b) Write two measures of mean historical returns. Calculate the arithmetic mean (AM) of annual holding yields of the investment : 1+2=3

Year	Beginning Value	Ending Value	HPY
1	100.0	115.0	0.15
2	115.0	138.0	0.20
3	138.0	110.4	-0.20

- (c) Calculate the risk in terms of variance and standard deviation of the investment in the following scenario :

3+2=5

<i>Economic Condition</i>	<i>Probability</i>	<i>Rate of Return</i>
Strong economy	0.15	0.20
Weak economy	0.15	- 0.20
No major change in economy	0.70	0.10

- (d) Discuss the following five risks : 5

(i) Business risk

(ii) Financial risk

(iii) Liquidity risk

(iv) Exchange rate risk

(v) Country risk of an investment

- (e) Define risk premium and systematic risk.

2+2=4

- (f) Write three ways to change the relationship between risk and the required rate of return for an investment.

4

Or

Write a short note on investment objective and investment constraints.

3. (a) Write two assumptions of the Markowitz's portfolio theory.

2

- (b) Find the variance and standard deviation of the following investment scenario :

4

Possible Rate of Return ( $R_i$ )	Expected Security Return $E(R_j)$	Probabilities ( $P_j$ )
0.08	0.103	0.35
0.10	0.103	0.30
0.12	0.103	0.20
0.14	0.103	0.15

- (c) Find the covariance of rates of returns of US stocks and US bonds as given below :

2010	US Stock Index ( $R_i$ )	US Bond Index ( $R_j$ )
January	- 3.60	1.58
February	3.10	0.40
March	6.03	- 0.85
April	1.58	1.05
May	- 7.99	1.71
June	- 5.24	1.87
July	7.01	0.68
August	- 4.51	2.01
September	8.92	0.02
October	3.81	- 0.16
November	0.01	0.70
December	6.68	- 1.80

If standard deviations of both scenarios are  $\sigma_i = 5.56$  and  $\sigma_j = 1.22$ , then find the correlation.

4+2=6

- (d) State and prove two-fund theorem. 5

Or

Write the assumptions of Capital Market theory.

- (e) State one-fund theorem. 2

- (f) Write short notes on any *two* of the following :  $3 \times 2 = 6$

(i) Optimal portfolio

(ii) Risk-free portfolio

(iii) Efficient frontier

4. (a) What are the values of—

(i) standard deviation of expected return of risk-free asset;

(ii) covariance of any two sets of returns of risk-free asset;

(iii) correlation between risky asset and risk-free asset?  $1 \times 3 = 3$

Or

Write a short note on Capital Market Line (CML).

3

- (b) Determine the expected rate of return with CAPM for the following five stocks :

Stock	Beta
A	0.70
B	1.00
C	1.15
D	1.40
E	-0.30

where economy's PER = 0.05 and expected return on the market portfolio  $E(R_M) = 0.09$ .

5

- (c) What is beta of a portfolio? Write the formula for beta of a portfolio. Interpret beta of 1.20 and 0.70. 2+1+2=5
- (d) What is security market line? How do you identify that an asset is properly valued, overvalued or undervalued on the graph of Security Market Line (SML)? 2+3=5

Or

Identify the following stocks which are properly valued, overvalued and undervalued :

5

Stock	Expected Return $E(R_i)$	Estimated Return
A	7.80	8.00
B	9.00	6.20
C	9.60	15.15
D	10.60	5.16
E	3.80	6.00

- (e) Suppose that during the most recent 10 years period the average annual total rate of return including dividends on an aggregate market portfolio was 14 percent ( $\bar{R}_M = 0.14$ ) and the average nominal rate of return on government T-bills was 8 per cent ( $\bar{RFR} = 0.08$ ). As administrator of a large pension fund that has been divided among three money managers during the past 10 years. Decide by calculating  $T$  values whether to renew their investment management contracts based on the following results :

Investment Manager	Average Annual Rate of Return	Beta
W	0.12	0.90
X	0.16	1.05
Y	0.18	1.20

Also plot their portfolios with security market line (SML).

7

Paper : DSE-1.3

( Financial Mathematics )

( For 2020 batch only )

1. (a) Let demand function of an item is represented by  $12q+15p=190$ . Write the inverse demand function. 1
- (b) Among demand and supply functions, write which function changes after introduction of excise tax. 1
- (c) Define equilibrium set for a market. 2
- (d) Define a first-order recurrence. 2
- (e) Describe intervals of compounding. 4

Or

Let supply and demand functions for an item are  $q^S(p) = bp - a$  and  $q^D(p) = c - dp$ . If an excise tax  $T$  per unit is imposed ( $T \neq 0$ ), then find the resulting market price  $p^T$ .

2. Answer any two from the following questions : 4×2=8

(a) Describe Cobweb model.

- (b) Let supply and demand sets for an item are

$$S = \{(q, p) : 2p - 3q = 12\}$$

$$D = \{(q, p) : 2p + q = 20\}$$

and initial price  $p_0 = 10$ . Find an expression for the price in the year  $t$ .

- (c) For the functions

$$S = \{(q, p) : q = bp - a\}$$

$$D = \{(q, p) : q = c - dp\}$$

describe stable and unstable market.

3. (a) Define revenue. 1
- (b) Write about inflexion point. 2
- (c) Let  $I(q) = -14 + 6q - 0.2q^2$  be the profit function of a firm which can produce 12 units per day. Find maximum profit. 5

Or

The supply and demand functions are defined by  $2q - 5p = 14$  and  $3q + 2p = 72$ . An excise tax  $T$  per unit is imposed. Determine when revenue will be maximum.

4. (a) Write when demand is called inelastic. 1
- (b) Define elasticity of demand. 2
- (c) Define startup point and breakeven point. 2+2=4
- (d) Explain competition versus monopoly. 5

Or

Let the demand is represented by  $q = ke^{-m}$ , where  $k$ ,  $m$  are constants. Explain elasticity.

5. (a) Explain the three cases how prices of two items may be related to each other. 4
- (b) Find and classify the critical points of

$$f(x, y) = x^3 - y^3 - 2xy + 1 \quad 6$$

Or

Find the maximum value of the function

$$f(x, y) = 6 + 4x - 3x^2 + 4y + 2xy - 3y^2$$

6. (a) Define arbitrage portfolio. 2
- (b) Answer any two from the following questions : 5×2=10

(i) Let

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \text{ and } A^n = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$$

Find recurrence equations for  $a_n$ ,  $b_n$ ,  $c_n$  and  $d_n$ .

- (ii) Describe technology matrix.
- (iii) Describe a two-industry economy.

7. (a) Define cash flow. 1
- (b) Define hedging. 1
- (c) Write about investment. 2
- (d) Describe comparison principle. 2
8. (a) Write the alternative name of interest. 1
- (b) Define effective interest rate. 1
- (c) Write True or False : 1  
Effective interest rate and nominal rate are same.
- (d) Write the relation between future value and present value. 2

- (e) Find the internal rate of return of the cash flow sequence (1, -1, 0, 1). 5

Or

Show that in simple interest, account grows linearly with time.

- (f) Describe municipal bonds and callable bonds. 4

Paper : DSE-1.3

( **Financial Mathematics** )

( For 2019 batch only )

UNIT—I

1. Answer the following as directed : 1×4=4

- (a) Write the alternative name of interest.  
(b) Define effective interest rate.  
(c) Effective interest rate and nominal rate are same.

( Write True or False )

- (d) Define discount factor.

2. Answer the following questions : 2×4=8

- (a) Write about investment.
- (b) Describe comparison principle.
- (c) Write risk aversion principle.
- (d) Define derivative asset.

3. Answer any *four* of the following questions : 6×4=24

- (a) Show that in simple interest, account grows linearly with time.
- (b) Show that for a cash flow stream  $(x_0, x_1, x_2, \dots, x_n)$  and an interest  $r$  per period the present value is

$$x_0 + \frac{x_1}{1+r} + \frac{x_2}{(1+r)^2} + \dots + \frac{x_n}{(1+r)^n}$$

- (c) Find the internal rate of return of the cash flow sequence  $(1, -1, 0, 1)$ .
- (d) Describe municipal bonds and callable bonds.
- (e) Write the uses and importance of hedging.

4. Describe comparison principle. 4

UNIT—II

5. Answer the following questions : 1×4=4
- (a) Define no-arbitrage assumption.
  - (b) Write the relation between future value and present value.
  - (c) Define annuity.
  - (d) Write when Jensen's index is zero.
6. Answer the following questions : 2×4=8
- (a) Write the risk aversion principle.
  - (b) Define derivative asset.
  - (c) Write two variations to the generic coupon bond.
  - (d) Write the linearity property of expected value.
7. Answer any *two* of the following questions : 4×2=8
- (a) Compute future value of cash flow stream (-1, 2, 1, 1.5), the periods are years and interest rate is 10%.
  - (b) Describe price yield curves.

( Turn Over )

(c) Describe Macaulay duration.

(d) Describe immunization.

8. Answer any *four* of the following questions :

5×4=20

(a) Describe three government securities.

(b) Find the corresponding effective rate for 3%, compounded monthly.

(c) Show that  $\frac{dp}{d\lambda} = -D_m P$  with usual notations.

(d) Describe the process of computing internal rate of return.

(e) Describe Markowitz model.

(f) State and describe capital asset pricing model.

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